Exam. Code : 103201
Subject Code : 1028

B.A./B.Sc. Ist Semester

MATHEMATICS

Paper—I (Algebra)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all, selecting at least **TWO** from each Section. All questions carry equal marks.

SECTION—A

1. (a) Reduce the matrix $\begin{bmatrix} 3 & -2 & 1 \\ 2 & -1 & 3 \\ 1 & -2 & 1 \end{bmatrix}$ to the form I_3 and find rank.

(b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ by elementary row operations.

2. (a) Determine whether the following matrices have same column space or not

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 15 \end{bmatrix}.$$

(b) Discuss for all values of K, the system of equations $(3K-8)x+3y+3z=0$, $3x+(3K-8)y+3z=0$, $3x+3y+(3K-8)z=0$.

3. (a) Examine the consistency of

$2x+3y+z=9$, $x+2y+3z=6$, $3x+y+2z=8$

If consistent, solve for x, y, z by finding the inverse of the coefficient matrix.

(b) Prove that the characteristic roots of a skew-hermitian matrix A are either purely imaginary or zero.

4. (a) Find the characteristic roots and the associated characteristic vectors for the matrix

$$\begin{bmatrix} -3 & -9 & -12 \\ 1 & 3 & 4 \\ 0 & 0 & 1 \end{bmatrix}.$$

(b) Verify Cayley-Hamilton theorem and find the inverse

of $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$.

5. (a) Find the characteristic equation and the minimal equation of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

Also show that A is non-derogatory.

(b) Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$.

SECTION—B

6. (a) Show that every positive definite or semi-definite matrix can be represented as gram matrix.

(b) Show that the form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ is indefinite and find two set of values of x_1, x_2, x_3 for which the form assumes positive and negative values.

7. (a) Solve the equation $32x^3 - 48x^2 + 22x - 3 = 0$, the roots being in A.P.

(b) Solve $3x^4 + 17x^3 - 5x^2 + 8x + 12 = 0$, given that the product of two roots is unity.

8. (a) Can the same transformation remove both the second and the fourth terms of $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$? If so, solve it completely.

(b) If α, β, γ are the roots of the cubic $x^3 - 3x + 1 = 0$, form an equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.

9. (a) If α, β, γ the roots of the equation $x^4 + 2x^3 + 3x^2 - x - 2 = 0$ find the value of $\sum \frac{\alpha\beta}{\gamma^2}$.

(b) Use Cardan's method to solve $x^3 - 3x^2 - 10x + 24 = 0$.

10. (a) Solve by Descartes' method

$$x^4 + 2x^3 - 7x^2 - 8x + 12 = 0.$$

(b) Use Ferrari's method to solve

$$x^4 - 5x^3 + 3x^2 + 2x + 8 = 0.$$